SIMILARITY OF UNDEVELOPED TURBULENT NEAR-WALL FLOWS

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It is shown that logarithmic formulas can be derived for the mean turbulent flow velocities at a smooth wall under lesser limiting assumptions than is customary by using the Isaacson–Milliken method. It can be concluded from a comparison with experiment that the generalized similarity laws are useful to describe turbulence at moderate Reynolds numbers.

In describing turbulent near-wall flows by using dimensional analysis and similarity, it is initially necessary to extract the subdomains with different governing parameters. Thus, for the flow near a smooth wall whose mean velocity can depend on the distance to the wall y, the boundary layer thickness (tube radius) δ, the dynamic velocity u_*, and the fluid viscosity ν (for a homogeneous incompressible fluid the density can be considered equal to one), two subdomains are ordinarily separated out [1]. As it is assumed, the dependence on δ is not essential in the internal subdomain near the wall, while the nature of the flow is independent of the viscosity in the external subdomain at the outer edge of the boundary layer. Correspondingly, we can write for the mean velocity distribution in these subdomains

$$\langle u \rangle = u_* f_1(y^+),\ y^+ \equiv y u_*/\nu; \tag{1}$$

$$U - \langle u \rangle = u_* f_2(\eta),\ \eta \equiv y/\delta. \tag{2}$$

Following Isaacson and Milliken [1], we assume that there exists a common domain in which the mean velocity distribution is representable in both form (1) and form (2) ("overlap domain"). The form of the dimensionless functions of the dimensionless variables $f_1(\eta)$ and $f_2(\eta)$ is determined to the accuracy of constant coefficients in this domain. Indeed, we can write for the mean velocity gradient in the overlap domain

$$y \frac{d}{dy} \langle u \rangle / u_* = y^+ f_1'(y^+) = -\eta f_2'(\eta),$$

and since the equality between functions of different arguments is possible only if they equal the same constant, it here follows

$$f_1(y^+) = A \ln y^+ + B_{10},\ f_2(\eta) = -A \ln \eta + B_{20}, \tag{3}$$

as well as an important formula for U/u_*

$$U/u_* = A \ln \xi + B_{10} + B_{20},\ \xi \equiv \delta u_*/\nu. \tag{4}$$

This is the traditional scheme for turbulent boundary-layer analysis which allows a simple generalization that turns out to be important for an inadequate development of the turbulent flow at a wall (for small Reynolds numbers). To derive the logarithmic distributions for the mean velocity by the Isaacson–Milliken method, weaker assumptions than those made above are adequate. It is sufficient to assume that cutting down the number of essential governing parameters is possible in just one of the subdomains, while all the parameters mentioned, y, δ, u_*, ν, play an important role in the other. Then, two cases are possible depending on which subdomain the reduction of the description holds.

Let us assume first that autonomy (independence of one of the parameters, namely δ) is valid just for the inner sublayer while the influence of viscosity is extended to the outer edge of the flow (to the axis in the case of a tube). Then, as before, (1) is true for the inner subdomain, but we can write in place of (2) for the outer part of the flow

$$U - \langle u \rangle = u_* F_2(\eta, \xi) = u_* F_2(y^+/\xi, \xi). \tag{5}$$

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Assuming that (1) and (5) can be used in some intermediate domain ($1 \ll y^+ \ll \xi$), we have the following relationship for the mean velocity gradient

$$y \frac{d}{dy} \langle u \rangle / u_* = y^+ f_1'(y^+) = - y^+ \frac{\partial}{\partial y^+} F_2(y^+/\xi, \xi), \tag{6}$$

in which an arbitrary* dimensionless function of one variable $A(y^+)$ can be written on the right in the general case, i.e., the logarithmic derivative of the mean velocity can depend on y^+ in the overlap domain. There follows from (6)

$$f_1(y^+) = \int dy^+ A(y^+)/y^+ + \text{const},$$

$$F_2(\eta, \xi) = - \int d\eta A(\eta\xi)/\eta + C_2(\xi).$$

If the function $A(y^+)$ is expanded† in a series of inverse powers of a large parameter

$$A(y^+) = A_{00} + A_{01} \frac{1}{y^+} + \ldots, \tag{7}$$

then we will have to the accuracy of terms of the three fundamental orders of magnitude

$$f_1(y^+) = A_{00} \ln y^+ + B_{10} - A_{01} \frac{1}{y^+} + \cdots, \tag{8}$$

$$F_2(\eta, \xi) = - A_{00} \ln \eta + B_2(\xi) + A_{01} \frac{1}{\eta\xi} + \cdots. \tag{9}$$

Taking account of (1) and (5), we hence obtain in place of (4)

$$U/u_* = A_{00} \ln \xi + B_{10} + B_2(\xi). \tag{10}$$

For this formula to be sufficiently meaningful, it is still necessary to determine the form of the function $B_2(\xi)$ from either additional considerations or experiment. It can be expected that for large Reynolds numbers ξ this function will tend to the universal constant B_{20} and (9) will go over into (4), and an expansion in the large parameter ξ

$$B_2(\xi) = B_{20} + B_{21} \frac{1}{\xi} + \cdots$$

can be used under conditions near the limit.

In conformity with (10), it is possible to express ξ in terms of the other large parameter U/u_* and to use an expansion in this parameter as in [2] to derive the drag formula for a boundary layer at low Reynolds numbers (changes eliminating the above-mentioned error should be inserted in these formulas).

For the other generalization we assume that, conversely, the outer subdomain remains autonomous, i.e., the outer flow is independent of the viscosity. As regards the inner subdomain, the influence of even the integral scale of type δ (or the radius in the case of a tube) on it is allowable. Then as before, (2) is valid for the outer subdomain and we obtain in place of (1)

$$\langle u \rangle = u_* F_1(y^+, \xi) = u_* F_1(\eta\xi, \xi). \tag{11}$$

In the overlap domain ($1/\xi \ll \eta \ll 1$), where (1) and (11) are simultaneously valid by assumption, we can write for the mean velocity gradient

$$y \frac{d}{dy} \langle u \rangle / u_* = \eta \frac{\partial}{\partial \eta} F_1(\eta\xi, \xi) = - \eta f_2'(\eta), \tag{12}$$

from which there follows analogously to the preceding

$$F_1(y^+, \xi) = \int dy^+ A(y^+/\xi)/y^+ + C_1(\xi);$$

$$f_2(\eta) = - \int d\eta A(\eta)/\eta + \text{const}. \tag{13}$$

*The same case with a nonautonomous outer sublayer was considered earlier in [2] and an unfounded conclusion on the constancy of the quantity A was made.

†The further analysis is close to that known for a thermally stratified boundary layer [1].

If the expansion of the function $A(\eta)$ in a series in the small parameter η is used

$$A(\eta) = A_{00} + A_{01}\eta + \cdots,$$

then the functions from (13) are also represented by the expansions

$$F_1(y^+, \xi) = A_{00}\ln y^+ + B_1(\xi) + A_{01}y^+/\xi + \cdots, \qquad f_2(\eta) = -A_{00}\ln\eta + B_{20} - A_{01}\eta + \cdots \tag{14}$$

We hence obtain a formula for U/u_*

$$U/u_* = A_{00}\ln\xi + B_1(\xi) + B_{20}, \tag{15}$$

which does not differ substantially from (10) for the case considered earlier. It actually yields the "local drag law" for the known universal function $B_1(\xi)$ and the universal constants A_{00}, B_{20} since $c_f = 2(u_*/U)^2$, $Re \equiv U\delta/\nu = \xi(2/c_f)^{1/2}$.

The essential distinction from the preceding case is that now namely the parameter B_1 of the inner similarity law (the "law of the wall") turns out to be a function of the Reynolds number ξ. A similar inconstancy of the additive coefficient of the logarithmic velocity profile at low Reynolds number has been repeatedly noted in experimental investigations. A great deal of attention has been paid to this fact in [3], in which a sufficiently strongly descending dependence of the coefficient B_1 on the integral Reynolds number is established for flow in tubes as the Reynolds number varies from $3\cdot 10^3$ to 10^4, and the coefficient B_1 reached the constant universal value B_{10} only for large Reynolds numbers.

For the case of a tube the dependence of B_1 on ξ is directly a dependence on a Reynolds number of the form ru_*/ν. However, it is easy to conceive that this is equivalent to the assertion about the dependence of B_1 on the Reynolds number understood in any other sense. Indeed, there follows from (15)

$$\frac{rU}{\nu} = \frac{U}{u_*}\frac{u_* r}{\nu} = \xi\,[A\ln\xi + B_1(\xi) + B_{20}],$$

so that ξ can be expressed in terms of rU/ν. Here U is the mean velocity on the tube axis, but U can be expressed in terms of the fluid mass flow rate by using integration of the velocity profiles (2) and (11) across the tube section.

As another proof of the absence of autonomy of the inner near-wall domain, the recently established experimental fact of the dependence of the mean period of "explosive" renewal of the flow in the viscous sublayer, i.e., in the inner domain, on such external factors as δ and U can be mentioned (see, e.g., [4]).

In conclusion, let us discuss still another possible generalization. We assume that in both the outer and inner subdomains there is autonomy (one is independent of the viscosity, and the other of the integrated length scale), however, still a third intermediate subdomain exists in which all the parameters y, δ, u_*, ν are important. Then as before, (1) and (2) will be applicable in the outer and inner subdomains while we can write

$$\langle u \rangle = u_*\varphi(y^+, \xi)$$

in the intermediate subdomain.

Assuming that there are two distinct overlap domains between adjacent subdomains, the form of the mean velocity profile in these overlap zones can be found completely analogously to the preceding

$$f(y^+) = A_{10}\ln y^+ + B_{10} - A_{11}\frac{1}{y^+} + \cdots,$$

$$\varphi(y^+, \xi) = \begin{cases} A_{10}\ln y^+ + B_{10} - A_{11}\dfrac{1}{y^+} + \cdots \\ A_{20}\ln y^+ + B_2(\xi) + A_{21}\dfrac{1}{\xi}y^+ + \ldots, \end{cases}$$

$$F(\eta) = -A_{20}\ln\eta + B_{20} - A_{21}\eta + \cdots$$

Attention is turned here to the appearance of two logarithmic sections of the velocity profile with different factors for the logarithms corresponding to the two distinct overlap domains.

NOTATION

y, distance to the smooth plane wall; δ, boundary layer thickness; u_*, dynamic velocity; ν, fluid viscosity; $\langle u \rangle$, mean flow velocity in the boundary layer; U, outer flow velocity; $y^+ \equiv yu_*/\nu$, dimensionless coordinate; f_1, F_1, f_2, F_2, notation for the universal dimensionless functions of the dimensionless arguments y^+; $\eta \equiv y/\delta$; $\xi \equiv \delta u_*/\nu$; A_{kl}, B_{mn}, numerical factors; $c_f = 2u_*^2/U^2$, local drag coefficient; $Re = U\delta/\nu$, Reynolds number.

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INFLUENCE OF TEMPERATURE ON THE HYDRODYNAMIC RESISTANCE REDUCTION EFFECT

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The effect of temperature on hydrodynamic resistance of aqueous solutions of polyethylene oxide is established experimentally.

Of the large number of studies dedicated to reduction in hydrodynamic resistance by addition of polymers, only in a few cases has the temperature dependence of this phenomenon been considered [1-4]. Analysis of these studies leads to the conclusion that the question of the effect of temperature on the effectiveness of polymer additives is still far from solved. At the same time clarification of this matter is not only of great practical interest, but would also permit a refinement of certain aspects of the Toms effect and a more detailed description of the structure of polymer solutions.

The materials studied in the present study were true aqueous solutions of polyethylene oxide (PEO), prepared by the firm WDN Chemicals, Ltd (England), having a molecular mass of $3 \cdot 10^6$. The hydrodynamic resistance was measured with the pumpless apparatus of [11], having the following basic parameters: working volume, $700 \cdot 10^{-6}$ m^3; tube diameter, $2.68 \cdot 10^{-3}$ m; channel length, 1.876 m. Measurement accuracy was of the order of 2%.

In the present study PEO solutions with a mass concentration of 0.003% were examined, this being the optimum concentration level at 18°C. This was done because the resistance reduction effect is more sensitive to changes in external conditions at concentrations equal to or less than the optimum one [3].

The experimental results shown in Fig. 1 indicate a decrease in effectiveness of the PEO additive with increase in temperature. It is also evident from the figure that at all temperatures studied, the transition from laminar to turbulent flow regime occurs at the critical Reynolds number, i.e., no protracted maintenance of laminar flow was observed. At 18 and 38.5°C the polymer additive begins to act effectively even in the transition region. At higher temperatures, after the transition to the turbulent regime, there exists a Reynolds number range in which the additive has no effect on flow resistance. The resistance reduction effect at these temperatures appears only after attainment of a threshold Reynolds number, the value of which increases with increase in temperature.

Since the studies were performed at elevated temperatures, it could be suggested that thermal destruction of the macromolecules had a significant effect on effectivenss of the polymer additive. This effect could not be avoided completely, so to reduce the effects of thermal destruction and improve repeatability of the results the solutions were maintained at elevated temperatures for identical times of 30 min. The following experiment was per-

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